June 2022

The First Iowa Math Tournament

sponsored by
MAA Dolciani Enrichment Grant and ICPL

General Instructions:

- Use the white spaces in each page or the provided scratch papers to do the calculations and mark the right answer in the multiple-choice answering-sheet with a pencil. Only return the answering sheet at the end.
- You have up to 120 minutes.
- Every of the 24 questions on the test has 6 points for each correct answer, for a maximum possible score of 144 points.
- Leaving a question blank points earns you 1.5 point and a wrong answer has 0 points; therefore, try not to randomly answer a question unless you are somewhat sure about your answer.
- No calculators are allowed (it is not really going to be useful anyway)
- It is probably hard for most people to go over all the 24 questions in the given time. So it is wise to not spend too much time on a single question and work on the questions that sound easy or familiar first.
- For any question, if you feel that the right answer is not in the list, write the correct answer next to the question number in the answer-sheet.

1. Evan has forgotten his room number. However, he remembers that his room was on the ninth floor of the hotel and that his room number had an odd number of prime factors. Strangely, he also remembers that his room number was not divisible by 3. What is Evan's room number?

    A. 900

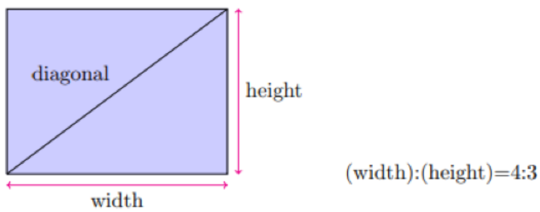
    B. 924

    C. 932

    D. 961

    E. 983

2. Many television screens are rectangles that are measured by the length of their diagonals. The ratio of the width to the height in an old standard television screen is 4 : 3. What is the width of an old 27-inch television screen, in inches?



    A. 30

    B. 21.6

    C. 27

    D. 16.2

    E. none of the above

3. By using all of the numbers from 1 to 9 we have made three 3-digit numbers. If the largest number is A, what is the smallest possible A?

    A. 345

    B. 198

    C. 912

    D. 398

    E. 312

4. Find the number of solutions of the equation

$$[x] + [-x] = -\left(\frac{x-1}{2}\right)^2$$

in the set of real numbers. Here, $[x]$ is the floor function in x (takes as input a real number x, and gives as output the greatest integer less than or equal to x).

   A. 0

   B. 1

   C. 2

   D. 3

   E. 4

5. What is the remainder of $2^{2^{64}}$ divided by $2^{64} - 1$?

   A. $2^{32} - 1$

   B. 3

   C. -1

   D. 1

   E. -3

6. What is the probability that two randomly selected distinct numbers out of $2, 3, 4, 5, 6$ and 7 are relatively prime?

   A. 3/5

   B. 2/3

   C. 11/15

   D. 4/5

   E. 13/15

7. With six colors, in how many ways we can color all six faces of a cube if we have to use all the colors ? Notice that two coloring schemes that differ by a rotation of the cube are considered the same.

   A. 24

   B. 30

   C. 120

   D. 360

   E. 720

8. The graph of a quadratic equation has vertex $(2, 4)$ and passes through the point $(3, 10)$. If the quadratic equation can be expressed as $f(x) = ax^2 + bx + c$, find the value of $a + b + c$.

   A. 4

   B. 6

   C. 10

   D. 12

   E. 16

9. Mary's goal is to score 8 or higher on AIME. For every hour she studies, her expected AIME score increases by 0.1. If Mary is guaranteed a score of 5 on AIME right now, at least how many more hours must she study to reach her goal?

   A. 5

   B. 25

   C. 30

   D. 50

   E. 80

10. In right triangle ABC with $\angle B = 90°$ and $\angle A = 20°$, let BM be the median from B to side AC. What is the $\angle BMC$?

    A. $10°$

    B. $20°$

    C. $40°$

    D. $60°$

    E. 80

11. A teacher has 20 identical apples to distribute among 5 students. Each student needs to have at least 1 apple but no more than 15 apples. In how many ways this can be done?

    A. $\binom{20}{5}$

    B. $\binom{19}{4}$

    C. $\binom{19}{4} - 5$

    D. $\binom{20}{4} - 5$

    E. none of the above is correct.

12. Dr. Tehrani is a very hard worker. He grades his students' geometry tests at a rate of x tests per hour. Yesterday, he realized if he works faster and grades the tests at a rate of $x + 2$ tests per hour, instead, he would get his work done in 2 less hours. If Dr. Tehrani works faster and grades 48 tests in a day, what is the value of x?

    A. 4

    B. 6

    C. 8

    D. 9

    E. 10

13. Compute the area of a trapezoid $ABCD$ where $AB||CD$, $AB = 12$, $BC = 20$, $CD = 37$, and $DA = 15$.

    A. 180

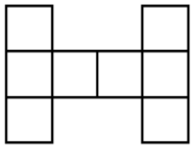
    B. 240

    C. 260

    D. 294

    E. 320

14. In how many ways we can fill the table below with $-1, -1, -1, 0, 0, 1, 1, 1$ such that the sum of the numbers in each (full) row, each (full) column, and four corners are the same?

    A. 1

    B. 12

    C. 16

    D. 24

    E. 48

15. Kevin has 18 cobs of corn, 6 identical ones of which are red, 6 identical ones of which are yellow, and 6 identical ones of which are purple. If Kevin wants to line up all his corn in a row such that no two red corn cobs are next to each other, how many ways are there for Kevin to line up his corn? (The following numbers are only the last 3 digits of the answer)

    A. 136

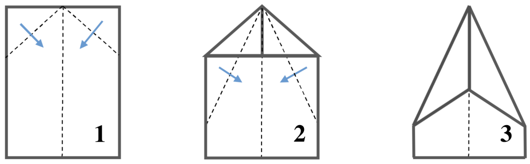
    B. 256

    C. 584

    D. 665

    E. 872

16. We fold a 6×8 paper multiple times as in the picture below. What is the area of the figure 3.



   A. $39 - 9\sqrt{2}$

   B. $38 - 8\sqrt{2}$

   C. $37 - 7\sqrt{2}$

   D. $36 - 6\sqrt{2}$

   E. $35 - 5\sqrt{2}$

17. The numbers 1 to 2022 are written on a white board. In each turn, we choose the largest number remaining on the board and erase all that numbers that divide it from the board, erasing them in order from the largest to the smallest. What is the last number that is erased?

   A. 1000

   B. 1010

   C. 1011

   D. 1012

   E. 1013

18. We have n natural numbers such that the difference every two of them is a prime number. What is the maximum possible value of n?

   A. 2

   B. 3

   C. 4

   D. 5

   E. 6

19. In a 4-gon $ABCD$ (i.e., a convex shape with 4 edges. For example a 3-gon means a triangle) there are 4 triangles (ABC, BCD, CDA, DAB) and each has 3 angles. If out of these 12 angles, n of them are obtuse, then

    A. n can not be 1

    B. n can not be 2

    C. n can not be 3

    D. n can not be 4

    E. none of above

20. In a village, there are 100 houses. Suppose there are at least 3 antennas over the roof of every 10 houses (some houses might have none and some others might have more than one), and you can find at least 2 crows on every 3 antennas (there could be more than one crown on a single antenna). At least how many crows live in this village?

    A. 10

    B. 92

    C. 93

    D. 98

    E. 99

21. Let r_1, r_2, and r_3 be the distinct roots of the polynomial $x^3 - 23x^2 + 142x - 120$. It is given that there exists real numbers p_1, p_2, and p_3 such that

$$\frac{1}{x^3 - 23x^2 + 142x - 120} = \frac{p_1}{x - r_1} + \frac{p_2}{x - r_2} + \frac{p_3}{x - r_3}.$$

    What is $\frac{1}{p_1} + \frac{1}{p_2} + \frac{1}{p_3}$?

    A. 386

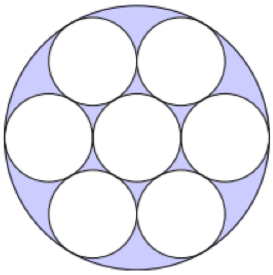
    B. 387

    C. 388

    D. 389

    E. 390

22. Each of the small circles in the figure below has radius one. The innermost circle is tangent to the six circles that surround it, and each of those circles is tangent to the large circle and to its small-circle neighbors. What is the area of the shaded region?



A. π

B. $4\pi/5$

C. $3\pi/2$

D. 2π

E. $7\pi/2$

23. Let a, b, c, d be the four roots of the degree four polynomial $x^4 - 5x^2 + 3x + 1$. What is the value of $a^4 + b^4 + c^4 + d^4$?

A. 40

B. 42

C. 44

D. 46

E. 48

24. In the triangle ABC we know $AB > AC > BC$. We rotate ABC a full turn around one of the edges to get a 3-dimensional object. Turning around which edge results in the maximum volume?

A. AB

B. AC

C. BC

D. The information is not sufficient to determine

E. The volume is the same for all three cases.